# A Subdivision Based Multi-color Separation Algorithm 

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#### Abstract

As the multi-color printing reproduces originals with more than four inks, the regular color separation methods are no longer suitable. The paper introduces a divisional method basing on primary color metric hue-angle which successfully applies the Neugebauer Equations to multi-color separation. According to this method, I divide the color separation process into two steps: Firstly, all the color is solved by divisional Neugebauer Equation to get the original separation data; then, substitute the black ink with cyan, magenta and yellow according to the gray balance curve. Every color of the original is reproduced by five primary colors finally, which solved the difficulty of the unsuitability of regular color separation methods. In additional, I make a difference between the gray color of the original and the gray component of the color, the two neutral gray is adjusted separately. And the experiments indicate that the method is effective.


## 0 Introduction

The chromatic digital originals, chromatic pictures or images, cover a widespread color gamut. It's impossible to yield all the color one color in one ink. Hence, the printing industry must utilize some kind of technique to synthesize all the color in certain kinds of inks, which is called color separation. For example, the regular four-ink printing takes cyan, magenta, yellow and black as its primary to synthesize kinds of color, and the multi-color printing takes more than four inks as its primary to synthesize, such as five, six or seven or even more. Color separation is a process that analyzes a color into several primaries. The color separation effect of an original can indicates the final corresponding reproduction. In a sense, the color separation effect determines the quality of the final reproduction.

The regular four-ink printing takes cyan, magenta, yellow and black as its primary to synthesize kinds of color, which can yield a biggish part of the gamut. However, the four-ink printing is limited by its gamut and insufficient degrees of freedom, it achieves, at best, a metameric reproduction [1], and the color is not saturated enough, while the utilization of two more inks can increase the color gamut and the number of degrees of freedom as well. Therefore, it enhances the possibility of colorimetric and spectral matching.

As mentioned above, the utilization of two more inks can enhance the resultant reproduction quality, however, it introduces another problem-how to make a color separation as it has more than four inks. As we all know, the regular four-ink printing utilizes Neugebauer equation to separate the color, it is no longer suitable for more than four inks, and we have to develop a new algorithm.

In this article, a divisional Neugebauer equation based sixcolor separation is proposed. As a matter of fact, there is a physical limitation that the total amounts of inks printed in a pixel printed with more than $400 \%$ dot area coverage will most likely collapse. This is a well-known phenomenon called the ink-trapping
limitation. Hence, it is not practical to print a color in all six inks. The proposed six-ink algorithm is to automatically select the black ink and other two inks.

## 1 The Ink Selection Algorithm

In this article, the proposed algorithm takes six inks of Cyan, Magenta, Yellow, Black, Blue and Orange as its primaries. The ink selection algorithm takes two steps: first of all, the metric lightness (L), metric chroma $\left(\mathrm{C}_{\mathrm{ab}}\right)$ and metric hue angle $\left(\mathrm{h}_{\mathrm{ab}}\right)$ of all the primary are calculated; and then they are described on the Metric hue-angle-Chroma plan (H-C plan).The coordinates points of every two neighboring primary of cyan, magenta, yellow, orange and blue combined with the that of black form five natural subdivisions, in other words, every subdivision is made up of the region consisting of black and other two neighboring primaries.

Practically, when a chromatic original is to separate, we just simply represent its color gamut in LCH color space. The projections on $\mathrm{H}-\mathrm{C}$ plan of these color points are supposed to fall into their corresponding subdivision, as for which subdivision they belong to relies on the relationship between their metric hue-angles and that of the primaries. For example, an arbitrary color P falls in the subdivision consisted by primaries magenta (M) and cyan (C), it is synthesized by magenta, cyan and black (Figure 1).

The boundaries of the subdivisions are determined by their metric hue-angles. We can calculate their metric hue-angles through the relationship between CIELAB and CIEXYZ. Its mathematical expression is as follows:

$$
\begin{equation*}
H_{a b}=\arctan \left(b^{*} / a^{*}\right) \tag{1}
\end{equation*}
$$

Where $\mathrm{H}_{\mathrm{ab}}$ is metric hue-angle, $b^{*}$ represents yellow-blue axis, and $a^{*}$ represents red-green axis.

Supposed that an arbitrary color has a metric hue-angle of $h_{\mathrm{p}}$, an arbitrary set of neighboring primaries has a set of metric hueangle of $h_{1}$ and $\mathrm{h}_{2}\left(h_{1}<h_{2}\right)$ :

If $h_{1} \leqslant h_{\mathrm{p}}<h_{2}$, then color p is synthesized by this set of primaries.

Notice that in the proposed algorithm, magenta has the biggest metric hue-angle and orange the smallest, when a color has a metric hue-angle bigger than that of magenta or smaller than that of orange, it is synthesized by magenta and orange


Figure 1. Subdivision sketch map


Figure 2. Neutral Color Yielding Curve

## 2 Neutral Grey Yielding Rules

Since a chromatic original covers both chromatic color and neutral grey color, and the chromatic color contains neutral grey component, the proposed algorithm distinguishes the two kinds of grey, they are yielded separately.

### 2.1 Neutral Grey Color in Original

The neutral grey color is yielded with the blackandwhite edition all alone. The blackandwhite edition dot area coverage (K) is mapped to the Lightness (L) according to the lightness curve that simulates the human vision. Just like the Figure2, the horizontal axis represents lightness with a range from 0 to 100 , and the vertical axis represents the blackandwhite edition dot area coverage ranged from 0 to 100 .

### 2.2 Neutral Grey Component of the Chromatic Color

The neutral grey component is synthesized by the divisional primaries.

## 3 Color Separation Algorithms

### 3.1 Algorithm Prototype

The proposed algorithm utilizes three-color Neugebauer equation [2]. What is different from the traditional Neugebauer equation is that the Neugebauer primaries are no longer the regular Cyan, Magenta and Yellow but the divisional primaries. The entire color gamut is divided into five subdivisions, thus there are five sets of divisional Neugebauer primaries (Table 1):

Table 1. Divisional Neugebauer Primary Cells

| KMO | KOY | KYG | KGC | KCM |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | K | K | K | K | K |
| 1 | M | O | Y | G | C |
| 2 | O | Y | G | C | M |
| 3 | KM | KO | KY | KG | KC |
| 4 | KO | KY | KG | KC | KM |
| 5 | OM | OY | YG | CG | CM |
| 6 | KOM | KOY | KYG | KGC | KCM |
| 7 | W | W | W | W | W |

Where each column represents a subdivision, and the eight cells right following the head represent the divisional Neugebauer primaries of the very subdivision.

The relationships between subdivisions and their corresponding primary cell dot area coverage are described as follows:

Table 2. The dot area coverage of subdivision KMO

| Divisional primary color <br> cell | Dot area overage | Symbol |
| :--- | :--- | :--- |
| K | $\mathrm{k}(1-\mathrm{o})(1-\mathrm{m})$ | $\mathrm{f}_{1}$ |
| M | $(1-\mathrm{k}) \mathrm{m}(1-\mathrm{o})$ | $\mathrm{f}_{2}$ |
| O | $(1-\mathrm{k})(1-\mathrm{m}) \mathrm{o}$ | $\mathrm{f}_{3}$ |
| KO | $\mathrm{ko}(1-\mathrm{m})$ | $\mathrm{f}_{4}$ |
| KM | $\mathrm{k}(1-\mathrm{o}) \mathrm{m}$ | $\mathrm{f}_{5}$ |
| OM | $(1-\mathrm{k}) \mathrm{om}$ | $\mathrm{f}_{6}$ |
| KOM | kom | $\mathrm{f}_{7}$ |
| W | $(1-\mathrm{k})(1-\mathrm{o})(1-\mathrm{m})$ | $\mathrm{f}_{8}$ |

Table 3. The dot area coverage of subdivision KOY

| Divisional primary color <br> cell | Dot area overage | Symbol |
| :--- | :--- | :--- |
| K | $\mathrm{k}(1-\mathrm{o})(1-\mathrm{y})$ | $\mathrm{f}_{1}$ |
| O | $(1-\mathrm{k}) \mathrm{o}(1-\mathrm{y})$ | $\mathrm{f}_{2}$ |
| Y | $(1-\mathrm{k})(1-\mathrm{o}) \mathrm{y}$ | $\mathrm{f}_{3}$ |
| KO | $\mathrm{ko}(1-\mathrm{y})$ | $\mathrm{f}_{4}$ |
| KY | $\mathrm{k}(1-\mathrm{y}) \mathrm{o}$ | $\mathrm{f}_{5}$ |
| OY | $(1-\mathrm{k}) \mathrm{oy}$ | $\mathrm{f}_{6}$ |
| KOY | koy | $\mathrm{f}_{7}$ |


| $W$ | $(1-k)(1-o)(1-y)$ | $f_{8}$ |
| :--- | :--- | :--- |

Table 4. The dot area coverage of subdivision KGY

| Divisional primary color <br> cell | Dot area overage | Symbol |
| :--- | :--- | :--- |
| K | $\mathrm{k}(1-\mathrm{g})(1-\mathrm{y})$ | $\mathrm{f}_{1}$ |
| G | $(1-\mathrm{k}) \mathrm{g}(1-\mathrm{y})$ | $\mathrm{f}_{2}$ |
| Y | $(1-\mathrm{k})(1-\mathrm{g}) \mathrm{y}$ | $\mathrm{f}_{3}$ |
| KG | $\mathrm{kg}(1-\mathrm{y})$ | $\mathrm{f}_{4}$ |
| KY | $\mathrm{k}(1-\mathrm{g}) \mathrm{y}$ | $\mathrm{f}_{5}$ |
| GY | $(1-\mathrm{k}) \mathrm{gy}$ | $\mathrm{f}_{6}$ |
| KGY | kgy | $\mathrm{f}_{7}$ |
| W | $(1-\mathrm{k})(1-\mathrm{y})(1-\mathrm{y})$ | $\mathrm{f}_{8}$ |

Table 5. The dot area coverage of subdivision KGC

| Divisional primary color <br> cell | Dot area overage | Symbol |
| :--- | :--- | :--- |
| K | $\mathrm{k}(1-\mathrm{c})(1-\mathrm{g})$ | $\mathrm{f}_{1}$ |
| C | $(1-\mathrm{k}) \mathrm{c}(1-\mathrm{g})$ | $\mathrm{f}_{2}$ |
| G | $(1-\mathrm{k})(1-\mathrm{c}) \mathrm{g}$ | $\mathrm{f}_{3}$ |
| KC | $\mathrm{kc}(1-\mathrm{g})$ | $\mathrm{f}_{4}$ |
| KG | $\mathrm{k}(1-\mathrm{c}) \mathrm{g}$ | $\mathrm{f}_{5}$ |
| CG | $(1-\mathrm{cg} \mathrm{cg}$ | $\mathrm{f}_{6}$ |
| KCG | kcg | $\mathrm{f}_{7}$ |
| W | $(1-\mathrm{k})(1-\mathrm{c})(1-\mathrm{g})$ | $\mathrm{f}_{8}$ |

Table 6. The dot area coverage of subdivision KCM

| Divisional primary color <br> cell | Dot area overage | Symbol |
| :--- | :--- | :--- |
| K | $\mathrm{k}(1-\mathrm{c})(1-\mathrm{m})$ | $\mathrm{f}_{1}$ |
| C | $(1-\mathrm{k}) \mathrm{m}(1-\mathrm{r})$ | $\mathrm{f}_{2}$ |
| M | $(1-\mathrm{k})(1-\mathrm{m}) \mathrm{r}$ | $\mathrm{f}_{3}$ |
| KC | $\mathrm{kc}(1-\mathrm{m})$ | $\mathrm{f}_{4}$ |
| KM | $\mathrm{k}(1-\mathrm{c}) \mathrm{m}$ | $\mathrm{f}_{5}$ |
| CM | $(1-\mathrm{k}) \mathrm{cm}$ | $\mathrm{f}_{6}$ |
| KCM | kcm | $\mathrm{f}_{7}$ |
| W | $(1-\mathrm{k})(1-\mathrm{c})(1-\mathrm{m})$ | $\mathrm{f}_{8}$ |

The divisional Neugebauer equation is as follows:

$$
\left\{\begin{array}{l}
X^{1 / n_{x}}=\sum_{i=1}^{8} f_{i} X_{i}^{1 / n_{x}}  \tag{2}\\
Y^{1 / n_{y}}=\sum_{i=1}^{8} f_{i} Y_{i}^{1 / n_{y}} \\
Z^{1 / n_{z}}=\sum_{i=1}^{8} f_{i} Z_{i}^{1 / n_{z}}
\end{array}\right.
$$

Where $\mathrm{X}, \mathrm{Y}$ and Z are the tristimulus of the destination color, $\mathrm{X}_{\mathrm{i}}, \mathrm{Y}_{\mathrm{i}}, \mathrm{Z}_{\mathrm{i}}$ represent the tristimulus of the divisional primaries, $f_{\mathrm{i}}$ is a function, which is described by Demichel equation, of the probable occurrence of the three primary inks, and the $n_{\mathrm{x}}, n_{\mathrm{y}}, n_{\mathrm{z}}$ is the modified Neugebauer equation factors.

### 3.2 Neugebauer Equation Solutions

Neugebauer equation, which is a non-linear equation, has no regular solutions. Practically, we usually utilize iterative methods or gradient methods [3].

Newton method is a method of iteration which has some advantages of high speed of convergence and simple formation. However, it requires an accurate initial value. Furthermore, in every iterative circle, it has to calculate $\mathrm{n}+\mathrm{n}^{2}$ times functional values and derivative values, the workload is extremely heavy.

Steepest descend method is a method of gradient which translates the non-linear problem solution into an optimal problem and solves it with an optimization method. This method is easy to calculate and converge, but its convergent speed, at a speed of linear convergence, is oppressive slow.

The proposed algorithm utilizes a compound methodutilizing the Steepest descend method to obtain a good initial value and then the Newton method to solve the equation.

### 3.3 Determination of the Modified Neugebauer Equation Factors

The Modified Neugebauer equation factors, which are close related to the printing circumstance, can not be calculated theoretically and measured by instruments practically. Hence, it has to be determined by experiment indirectly.

Once a set of color is designed at given dot area coverage ( $a_{i}$ , $\beta_{\mathrm{i}}, \gamma_{\mathrm{i}}$ ) and printed under certain circumstance, the corresponding tristimulus ( $X_{\mathrm{i}}, Y_{\mathrm{i}}, Z_{\mathrm{i}}$ ) can be measured. In the formula $2, f_{\mathrm{i}}$ can be calculated according to the Demichel equation, and then the problem is translated into a problem of errors compensation with redundant observations, we can solve it using least square method [4].

## 4 Algorithm Programming

First of all, once a given chromatic original is selected, it is done gamut mapping in CIELAB color gamut, and then all its color pixel points are processed one by one:
(1) For chroma, if $a^{*}=b^{*}=0$, then the color is of neutral grey color, and it is yielded through neutral grey color yielding curve as mentioned above;

Otherwise, the color is of chromatic color, and then calculates its metric hue-angle and corresponding three primaries' dot area coverage according to its metric hue-angle. As all the other inks don't contribute to the very color synthesize, they are set to 0 , which means no dot at the pixel;
(2) Calculate the corresponding cyan, magenta and yellow dot area coverage under every blackandwhite edition dot area coverage according to the grey balance cure; Replace the black ink with a compound of cyan, magenta and yellow in terms of the replacement ratio.
(3) Final dot area coverage

The final dot area coverage composes of two parts: the calculated values from the divisional Neugebauer equation and the correction amounts from the grey balance curve and replacement ratio.

For example, some color is supposed to be synthesized by green, cyan and black, its calculated values from the divisional Neugebauer equation are $a_{g b} a_{0}$ and $a_{k j} ;$ the corresponding correction are $\Delta \mathrm{a}_{\mathrm{c}}, \Delta \mathrm{a}_{\mathrm{m}}$ and $\triangle \mathrm{a}_{\mathrm{y}}$, then the final dot area coverage are as follows:
$a_{c}=a_{c 0}+\triangle a_{c}$,
$\mathrm{a}_{\mathrm{m}}=\mathrm{a}_{\mathrm{m} 0}+\triangle \mathrm{a}_{\mathrm{m}}$,
$a_{y}=\triangle a_{y}$,
$a_{k}=a_{k 0}-\triangle a_{k}$
$\mathrm{a}_{\mathrm{g}}=\mathrm{a}_{\mathrm{g} 0}$
In this example, the color is to be synthesized by cyan, magenta, yellow, black and green at their dot area coverage respectively.

Notice that when all the pixels of the original have been processed, the procedure is over.

## 5 Conclusion

In order to verify the correctness and feasibility of the proposed algorithm, an experiment is carried out, and the experimental result indicates that the proposed algorithm in this
article is correct and feasible.
In this article, the traditional Neugebauer equation is introduced into the multi-color separation, which brings forward an important idea and feasible method for the multi-color separation researches yet to come. In some sense, it has an instructive significance for the multi-color separation.

## Reference

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## Author Biography

Cao Zhaohui received his bachelor's degree from the He 'Nan University (2004) and his master's degree from the Institute of Surveying and mapping, Information Engineering University (2007), and now he is reading for his PhD in the Institute of Surveying and mapping, Information Engineering University. Since he read for his bachelor's degree, his work has focused on the development of digital printing.

